EFFECT OF UNCERTAINTY IN COORDINATES OF THERMOCOUPLE LOCATION ON THE QUALITY OF SOLUTION OF THE BOUNDARY-LAYER INVERSE PROBLEM OF HEAT EXCHANGE

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The dependence of the reconstruction error of the thermal flux density on the boundary of a body on the accuracy of given coordinates of a temperature detector located inside the object is investigated.

Analysis of heat-exchange processes is often accompanied with the necessity of performing experimental studies, making it possible to obtain verifiable information on the thermal state of objects. The use of methods of inverse problems of heat exchange (IPHE), making it possible to reconstruct the thermal parameters from their causal manifestations, the temperatures at some points of the body, makes it possible to enhance substantially the informativity of the experiments [1]. Methods currently developed of solving incorrect problems [1, 2], including, among others, IPHE, are effectively used in thermal projection. Nevertheless, there are a number of unsolved problems in this area. One of them is related to the inadequate choice of mathematical models of thermal processes or their parameters due to the distorting action of the measuring apparatus. In particular, for thermocouple temperature measurements the main error sources include uncertainty in the material structure along the thermoelectrodes, plastic deformation of the thermoelectrodes, the Peltier, Thomson, and Joule effects, electric and magnetic focusing, variations in resistance of the thermocouple chain [3], inaccuracies in calibration characteristics, nonquality preparation of thermocouple junctions and nonreliable thermal contact of the thermocouple with the investigated material, heat removal from the electrodes, and the presence of a foreign body (thermocouple) inside the material [4]. According to [3], in many cases the most substantial effect on the accuracy of temperature measurements, particularly nonstationary ones, is due to the last two error sources.

Under these conditions, in solving inverse problems it is advisable to model the thermocouple by a body of finite sizes with certain geometric and thermophysical characteristics [5]. Often, however, the use of similar models is not justified, since it leads to substantial complication in the algorithms of IPHE solutions and a sharp increase in computing time. In such cases there is no alternative but represent the thermocouple by a finite body, with its location assumed known with some error. Thus, to decrease the error during temperature measurements in thermal protection coatings the thermocouples are placed in isothermal planes. In this case the uncertainty in the coordinate of the measurement point is approximately $\pm R$, where R is the radius of the electrode [4]. Since the temperature drop in this portion can be substantial (this is valid, for example, for materials with low thermal conductivity), the following problem is generated: to estimate the reliability of reconstructing thermal parameters from IPHE solutions due to errors in selecting coordinates of the measurement detector. In the present paper we provide study results of the sensitivity function of the thermal flux density $q(\tau)$ to coordinate variations in the thermocouple position for some input temperature, and using it in the interpretation of results of solving the one-dimensional boundary-value IPHE.

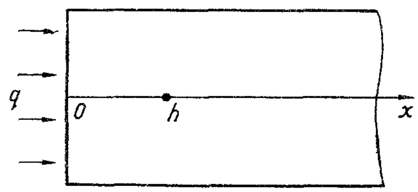

Fig. 1. Thermocouple position inside the material.

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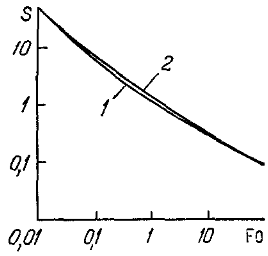

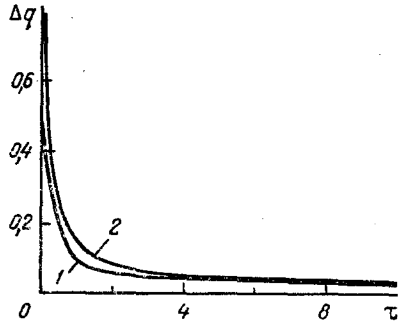

Fig. 2   Fig. 3

Fig. 2. Sensitivity function as a function of the Fourier number: 1) by Eq. (4); 2) by Eq. (5).

Fig. 3. Absolute error time distribution of reconstructed thermal flux density ($\Delta h = 0.1 \cdot 10^{-3}$ m): 1) result of IPHE solution; 2) calculation by the use of Eq. (5). Δq, 10^6 W/m^2; τ, sec.

To analyze the effect of the coordinate of the thermocouple position on the quantity q we use the one-dimensional model of a semiinfinite body (Fig. 1) with constant thermophysical characteristics, which in many cases describes quite accurately the heat propagation process.

As is well known, for $q(\tau)$ = const the nonstationary temperature distribution in the semiinfinite body is described by the expression [6]:

$$T(x,\ \tau) = \frac{q}{\lambda}\left[2\sqrt{\frac{a\tau}{\pi}}\exp\left(-\frac{x^2}{4a\tau}\right) - x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{a\tau}}\right)\right] + T_0. \qquad (1)$$

For simplicity we assume $T_0 = 0$. At the thermocouple position h we then have

$$T^*(\tau) = \frac{q}{\lambda}\left[2\sqrt{\frac{a\tau}{\pi}}\exp\left(-\frac{h^2}{4a\tau}\right) - h\,\mathrm{erfc}\left(\frac{h}{2\sqrt{a\tau}}\right)\right]. \qquad (2)$$

The boundary-value IPHE consists of seeking a density of thermal flux $q(\tau)$ for input temperature $T^*(\tau)$ and the value h. Accordingly, to determine the sensitivity function of the quantity q to a variation in the parameter h we differentiate (2) with respect to h, assuming that T^* is independent of h. As a result we then obtain the equality

$$q_h'\left[2\sqrt{\frac{a\tau}{\pi}}\exp\left(-\frac{h^2}{4a\tau}\right) - h\,\mathrm{erfc}\left(\frac{h}{2\sqrt{a\tau}}\right)\right] - q\,\mathrm{erfc}\left(\frac{h}{2\sqrt{a\tau}}\right) = 0, \qquad (3)$$

which can be rewritten as follows:

$$S(\mathrm{Fo}) = \frac{h}{q}\,q_h' = 1\Bigg/\left(\frac{2\sqrt{\mathrm{Fo}}\exp\left(-\frac{1}{4\mathrm{Fo}}\right)}{\sqrt{\pi}\,\mathrm{erfc}\left(\frac{1}{2\sqrt{\mathrm{Fo}}}\right)} - 1\right), \qquad (4)$$

where S is the sensitivity function of q to a variation in h, and $\mathrm{Fo} = a\tau/h^2$ is the Fourier number.

It must be noted that within 20% one can use for S the simpler expression

$$S(\mathrm{Fo}) = \frac{1}{2}\left(\frac{1}{\mathrm{Fo}} + \sqrt{\frac{\pi}{\mathrm{Fo}}}\right), \qquad (5)$$

coinciding with (4) for $\mathrm{Fo} \to 0$ and $\mathrm{Fo} \to \infty$ (Fig. 2).

Strictly speaking, the assumption of independence of the temperature $T^*(\tau)$ on h is satisfied only approximately, since if there exists a set (q, h), where q = const, guarantee-

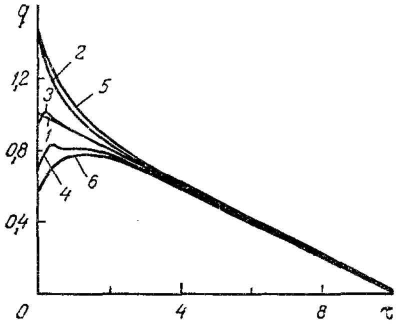

Fig. 4. Time dependence of the thermal flux density: 1) real thermal flux density; 2, 3, 4) reconstructed from the IPHE solution for $h = 0.9 \cdot 10^{-3}$, 10^{-3}, and $1.1 \cdot 10^{-3}$, respectively; 5, 6) calculated by the use of Eq. (8) for $\Delta h = -0.1 \cdot 10^{-3}$ and $0.1 \cdot 10^{-3}$ m, respectively. q, 10^6 W/m².

ing the temperature $T^*(\tau)$ at the point h, it is unique. In other words, the set $\{T^*(\tau), h_1\}$, where $h_1 \neq h$, corresponds to a nonconstant thermal flux density $q_1(\tau)$ as a consequence of the nonlinearity of expression (2) in the parameter h. Due to this dependence Eq. (4) is approximate. Nevertheless, as shown below, it describes quite accurately the real situation.

Thus, if the coordinate h is known with an uncertainty $\pm\Delta h$, the thermal flux density, reconstructed from the IPHE solution, possesses an absolute error $\pm\Delta q$:

$$q(\tau) = q_0(\tau) \pm \Delta q(\tau), \tag{6}$$

where $q_0(\tau)$ is the reconstructed thermal flux density for an accurate choice of the coordinate h.

Retaining the zeroth and first term in the expansion of the quantity $q(\tau)$ in a power series in the parameter $\Delta h/h$, we obtain an expression for the reconstruction error:

$$\Delta q(\tau) = q_0(\tau) S\left(\frac{a\tau}{h^2}\right)\frac{\Delta h}{h}. \tag{7}$$

Below we provide results of solving the IPHE model for a semiinfinite body with the thermophysical characteristics $a = 10^{-6}$ m²/sec, $\lambda = 1$ W/(m·K) and the parameter values $h = 10^{-3}$ m, $\Delta h = 0.1 \cdot 10^{-3}$ m, $\tau \in [0, \tau_m]$, $\tau_m = 10$ sec. The choice of the geometric parameters corresponds to the situation, when a thermocouple of diameter 0.2 mm is located at a distance of 1 mm from the heating surface.

The IPHE solution was carried out numerically by an iteration method [1], and the temperature field was represented in the form of a Duhamel convolution [6] with $n_\tau = 50$, where n_τ is the number of time discretization steps. The computer time for calculating one IPHE was 20 min on an ES-1022 computer.

Figure 3 shows the error Δq in reconstructing the thermal flux density, caused by an uncertainty in the thermocouple coordinate $\Delta h = 0.1 \cdot 10^{-3}$ m. As input information for the IPHE we selected the solution of the direct problem of thermal conductivity with the boundary condition $q(\tau) = 10^6$ W/m². In the same figure we show the time distribution of the error, calculated by Eqs. (5), (7). It is seen that almost at all times, except the initial portion, calculation of the error with the use of expression (5) for the sensitivity function provides satisfactory results. At short times there is a divergence, due to the fact that for $\mathrm{Fo} \to 0$ $S(\mathrm{Fo}) \to \infty$. With account of this it is advisable to correct the shape of the sensitivity function as follows:

$$S(\mathrm{Fo}) = \frac{1}{2}\left(\frac{1}{\mathrm{Fo}+0{,}2} + \sqrt{\frac{\pi}{\mathrm{Fo}+0{,}2}}\right). \tag{8}$$

Figure 4 shows the results of reconstructing the thermal flux density for the input temperature, obtained from solving the direct problem with $q(\tau) = 10^6 \cdot (1 - 0.1\tau)$ W/m². It is seen that the inaccuracy in the given coordinate of the temperature measurement point causes an error in the reconstructed function at the initial phase, while at long times the reconstruction occurs relatively accurately.

Thus, the use of the sensitivity function (8) makes it possible to estimate effectively the error of IPHE solutions, related to the uncertainty in the coordinate of the thermocouple position.

NOTATION

x and τ, spatial and time coordinate, respectively; τ_m, observation time; $T(x, \tau)$, temperature field; h, depth of thermocouple mounting; $q(\tau)$, thermal flux density supplied to the boundary of the body; a, thermal diffusivity coefficient; and λ, thermal conductivity coefficient.

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AN INVESTIGATION OF HEAT EXCHANGE IN THE HUMAN ORGANISM ON EXPOSURE TO INTERNAL HEAT

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The thermal field of a person exposed to artificial heat is investigated. A variation in the blood flow as the temperature inside the body increases is calculated by solving the inverse problem of heat conductivity.

A disturbance of physiological processes in the human organism is often followed by changes in the thermal regime of the body; therefore, characteristics of the thermal field of the body can be used as indices of the physiological state in the diagnosis of disease and in the course of treatment [1]. At present, thermovisual (thermographic) methods of diagnosis based on variations of the thermal field of the skin in the presence of pathologic foci in the internal organs [2] are getting wide recognition.

Thermographical investigations in medicine are basically characterized by a symptomatic approach: a diagnosis is made on the basis of a subjective interpretation of a purely visual picture of a thermographical image. Biophysical mechanisms of the formation of a thermal field on the surface of the body for the majority of pathologic violations in the internal organs have still not yet been studied. For example, one of the most important issues of a thermovisual diagnosis still remains unclear: What is the cause of the change in the temperature of the skin in one or other case, a thermal flow penetrating through the "shell" from the internal organs or reflective changes in the blood flow and heat generated in the surface tissues [1]?

In order to find objective criteria and to define areas for suitable application of thermographical methods of diagnosis, theoretical and experimental investigations of the processes of heat exchange in the human organism are conducted. In [3, 4], a mathematical model is proposed, which allows one to calculate the thermal field of the skin, given the values of heat productivity and blood flow in different organs and tissues of the body. However, quantitative information about thermophysical properties of tissues, about the distribution of sources of heat, and the consumption of blood in the internal organs and tissues of the "shell" for normal and pathological cases is specified with a great error. This is due both to the individual spread of parameters and to reflective reactions, in-

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